Comparative Study of Non-Uniform Rayleigh Beam with Variable Axial Force Resting on Bi-Parametric Foundation under Constant distribution and Moving point Loads

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Abstract

Comparative study of non-uniform Rayleigh beam with variable axial force resting on bi-parametric elastic foundations under the actions of moving constant distribution and distributed loads was investigated in this research. In order to obtain the solutions to the fourth order non-homogeneous partial differential equation governing the dynamical problem, the Galerkin Method (GM) was first used to reduce the fourth order partial differential equation into a second order ordinary differential equation. Then, the resulted equation was further treated using the Laplace transform method in conjunction with Convolution theorem to obtain the analytical approximate solutions. Finally, the Maple computer simulation software was used to present the results of the transverse displacement response to the non-uniform Rayleigh beam under the actions of moving constant distribution and distributed loads. The results in plotted curves reveals that for both moving constant distribution and distributed loads, the deflection profiles of the beam decreases as the values of the structural parameters such as axial force (N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc), and rotatory inertia correction factor (R_o) increases. Also, the deflection profiles of non-uniform Rayleigh beam under the actions of moving distributed loads is higher than that under the actions of moving constant distribution loads.

Keywords: Parametric Foundation, Galerkin Method, Rayleigh Beam, Variable Axial Force

1. Introduction

The concept of structural dynamics deals with moving loads on different structural members, studies in this area are enormous and have been of interest to Engineers, Applied Mathematicians and Applied Physicists. The problem has been the major concern of many researchers (e.g. Shadnam and Akin, 2001; Oni and Awodola, 2003; Oni and Ogunyebi, 2008). Its applications relate to the design, control, diagnosis, as well as life management of structural systems (e.g. highways, railways, bridges, cable-railways, overhead cranes and the likes) which are traversed by one or more moving loads.

Generally, emphasis is placed on the structural members and systems rather than on that of the moving loads. Common examples of structural members include beams, plates and shells while travelling loads includes trains, trucks, cars, bicycles, cranes and so on. Structural members may be classified into elastic, non-elastic or visco-elastic. When structural members or systems are acted upon by moving loads, the interaction between the moving loads and the structural members or systems makes the dynamic response analysis very complex. While constant stationary loads produce reactions, stresses and deformation that are constant, moving loads produces effects that are variable functions of their positions, which is also a function of time (Abiola and Gbadeyan, 2015; Oni and Ogunyebi, 2008). The construction of sophisticated structural systems in the early part of the 19th century drew the attention of many researchers to this area of study as the problem of moving loads exist in a wide variety of applications such as in the automotive and aircraft braking systems, conveyer systems, rail mechanics, noise from spinning floppy and hard discs of computer, belt drives (carrying machine chains), bridge or ground excitations from travelling vehicles and also in the vibrations produced by machining processes from different manufacturing industries.

The deflections of structural members are of considerable interest to the engineers designing mechanical and structural systems. Often times, artificial stresses are created in structures before loading, so that the stresses which then exist in the structures under the actions of moving loads are more favorable than would otherwise be the case. These artificial stresses are forces which may act axially or otherwise. If they are acting axially, they are called axial force. The artificial stresses are also called pre-stress. The aim of pre-stress in structures is to limit tensile stresses and hence flexural cracking or bending in the structure under working conditions, it is to external deformation and hence bending deformation. (Jimoh *et al*., 2017) if $N(x)$ is the variable axial force and $Y(x,t)$ is the axial deflection, the additional term is given by: $\frac{\partial}{\partial x}\left\{N(x)\frac{\partial Y(x,t)}{\partial x}\right\}$ (1)

for a pre-stress that varies with partial coordinates and also becomes: $N_o\frac{\partial^2 Y(x,t)}{\partial x^2}$ (2)

for a constant pre-stress

2. Literature Review

The problem of structural dynamics and response of solid elastic bodies (beams, plates or shells) under the actions of moving loads has been the major concern of many researchers in the field of Engineering, Applied Mathematics and Applied Physics. Thus, by virtue of its relevance to the design, control, diagnosis, as well as life management of structural systems (e.g. highways, railways, bridges, cable-railways, overhead cranes and the likes) traversed by one or more travelling loads (e.g. trains, trucks, cars, bicycles, cranes etc.), studies in this field shows an extensive investigation on a number of experimental and numerical problems reported in literature over the years. A review of some previous related works is as follows:

Ramin and Wan (2014) determined the natural frequencies and mode shapes of the axially loaded double structural member system consisting of two homogeneous and prismatic beams with a distributed spring in parallel between them using the exact dynamic stiffness method. The effects of structural parameters such as axial force, shear modulus and rotatory inertia were considered. The Laplace transform method was used to formulate the dynamic stiffness influence coefficients from the governing differential equations

Ogunyebi and Adedowole (2017) studied the dynamic behavior of uniform Rayleigh with an accelerating distributed mass. The technique adopted for the uniform forced vibration of the Rayleigh beam with accelerating mass resting on variable elastic foundation at constant velocity is Fourier sine transform and Laplace transform methods. The dynamic effect of vital parameters such as elastic foundation stiffness, rotatory inertia correction factor, and axial force were obtained.

Jimoh and Ajoge (2018) investigated the effects of rotatory inertia correction factor and damping coefficient on the transverse motion of uniform Rayleigh beam under moving load of constant magnitude. The solution technique was based on the Fourier sine integral transform method, Laplace transform method and convolution theorem. It was observed that the amplitude of the deflection of the beam under the influence of moving load of constant magnitude decreases with an increase in the values of rotatory inertia and damping coefficient and also that the effect of rotatory inertia correction factor is significant compared to that of damping coefficient.

Oni and Ogunyebi (2018) studied the dynamic response of uniform Rayleigh beams on variable bi-parametric elastic foundation under partially distributed loads. The dynamical system is governed by a fourth order partial differential equation with variable and singular coefficients which was solved using the Generalized Galerkin Method (GGM) and a modification of the Struble's asymptotic technique. In this study, it was confirmed that the moving distributed force is not a safe approximation to the moving distributed mass problem.

3. Methodology

In order to analyze the dynamic behavior of the beam under consideration our concern is to find the solutions to the transverse displacement, $Y(x,t)$ (3)
of the beam using the Galerkin Method (GM) in conjunction with Laplace transform method and convolution theorem.

The Galerkin Method (GM)

The exact solutions to the governing equations are generally not available. Hence, numerical methods are introduced to obtain approximate solutions. In solving nonlinear partial differential equations, we often apply approximation. One of the approximation methods: Galerkin's method, invented by a Russian mathematician named Boris Grigoryevich Galerkin. The Galerkin's method is a weighted residual method (WRM). In weighted residual methods, the integral of the residual (or coordinates in modal space), $q_j(t)$ multiplied by a weighting function (or mode shape of free vibration of the beam), $\beta_j(x)$is equated to zero. The mode shape functions are chosen to be the same form as each part of the approximate solution.

$$\int_0^L q_j(t)\,\beta_j(x) = 0 \qquad , \qquad j = 1,2,\dots,n \tag{4}$$

Where the residual (or coordinates in modal space), $q_j(t)$ is substituted into the approximate solution given as: $Y(x,t) = \sum_{j=1}^{\infty} q_j(t)\beta_j(x)$, $j = 1,2,\ldots$ (5)

Laplace Transform Method

This is an integral transform method that changes a real variable function $f(t)$ into a function $F(s)$ of complex variables. This method is stated thus: Let $f(t)$ be defined for $0 \leq t < \infty$. Then, when the improper integral exists, the Laplace transform $F(s)$ of $f(t)$, written as:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t)dt \tag{6}$$

Where, '$\mathcal{L}$' and 's' are called the Laplace transform operator and variable, while the functions $f(t)$ and $F(s)$ are the Laplace transform pair, and for all ordinary functions, given $F(s)$ the corresponding function $f(t)$ is determined uniquely just as $f(t)$ determines $F(s)$ uniquely.

Convolution Theorem

Let the functions $\boldsymbol{f(t)}$ and $\boldsymbol{g(t)}$ be defined for $\boldsymbol{t \geq 0}$. Then the convolution of the functions $\boldsymbol{f}$ and $\boldsymbol{g}$ denoted by $(\boldsymbol{f * g})(\boldsymbol{t})$, and in abbreviated form by $(\boldsymbol{f * g})$, is defined as the integral: $(f * g)(t) = \int_0^t f(u)g(t-u)du$ (7)

The change of variable $v = t - u$ followed by replacement of the dummy variable v by t shows that convolution operation is commutative, so $(f * g)(t) = (g * f)(t)$ (8)

Solution to the Moving Constant distribution Load Problem

$$Y_j(x,t) = \sum_{j=1}^{\infty}\left(\frac{Q\beta_o}{{\beta_o}^2+{n_1}^2}(e^{-n_1 t} - cos\beta_o t) - \frac{Qn_1}{{\beta_o}^2+{n_1}^2} sin\beta_o t - \frac{Q\beta_o}{{\beta_o}^2+{n_2}^2}(e^{-n_2 t} - cos\beta_o t) + \frac{Qn_2}{{\beta_o}^2+{n_2}^2} sin\beta_o t\right) sin\frac{j\pi x}{L} \tag{9}$$

Solution to the Moving Distributed Load Problem

$$Y_j(x,t) = \sum_{j=1}^{\infty}(TT_1(e^{r_1 t} - cos\beta_o t) - TT_2(e^{r_2 t} - cos\beta_o t) + T(T_3 - T_4)sin\beta_o t + TT_5(e^{r_1 t} - 1) - TT_6(e^{r_2 t} - 1))sin\frac{j\pi x}{L} \tag{10}$$

4. Results and Discussion

In order to illustrate the analysis in view, a Rayleigh beam of length, $L = 12.192m$ was considered, the beam is carrying a load of mass, $M = 8407.28\text{kgm}^{-1}$ moving with a constant velocity, $c = 8.128\text{ms}^{-1}$. The modulus of elasticity of the beam, $E = 2.109 * 10^9 Kgm^{-2}$, the moment of inertia of the cross section of the beam, $I_o = 2.87698 * 10^{-3}\text{m}^4$, the mass per unit length of the beam, $\mu_o = 4501.563Kgm^{-1}$, the values of the axial force (N_o), is varied between $(0N \ and \ 200000000N)$, the values of the foundation stiffness (K_o), is varied between $(0Nm^{-3} \ and \ 400000000Nm^{-3})$, the values of the shear modulus (F_o), is varied between $(0Nm^{-3} \ and \ 900000000Nm^{-3})$, the values of the damping coefficient (Δc), is varied between $(0 \ and \ 1000000)$, the values of the rotatory inertia correction factor (R_o), is varied between $(0 \ and \ 100)$, the value of pie, $\pi = \frac{22}{7}$, and the gravitational acceleration, $g = 9.81\text{ms}^{-2}$.

For the simply-supported non-uniform Rayleigh beam resting on a functionally graded constant bi-parametric elastic sub-grade, the graphical results to the analytical approximate solutions, $Y_j(x,t)'s$ of the beam under the actions of both moving constant distribution and distributed loads with the effects of structural parameters such as axial force (N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc), and rotatory inertia correction factor (R_o) are illustrated in plotted curves against time, t as shown below;

All main headings should be bold and use capital-initials. Sub headings should not be bold but in italics. Capital-initials should also be used for sub headings.

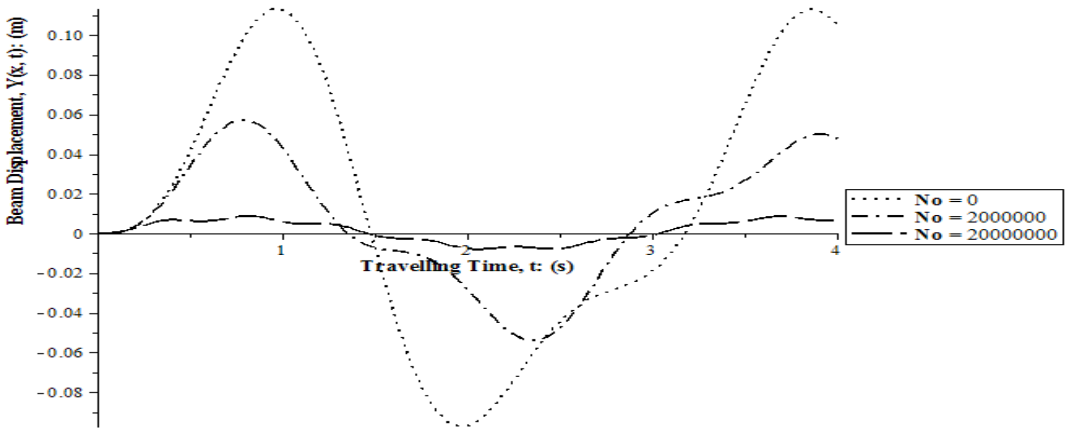

Figure 4.1a: *Deflection profile of non-uniform Rayleigh beam under moving constant distribution loads for various values of axial force (N_o) and fixed values of foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc) and rotatory inertia correction factor (R_o).*

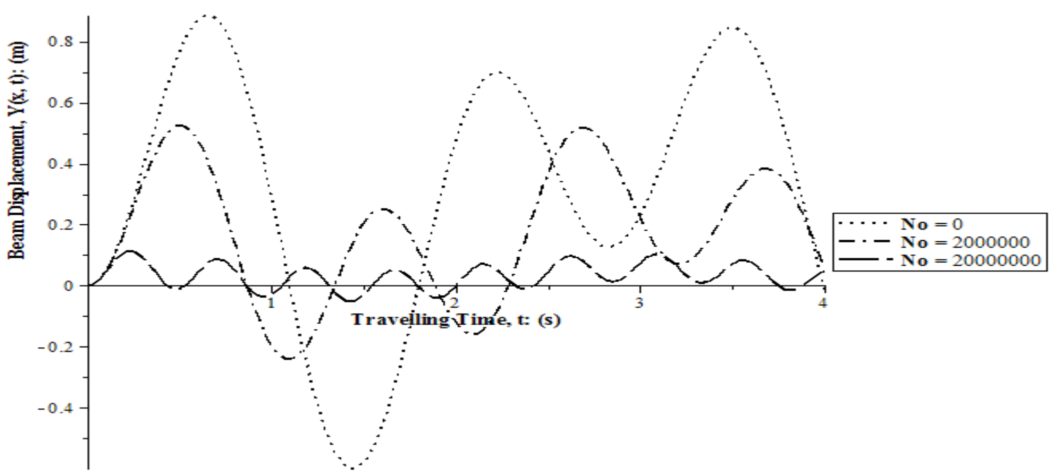

Figure 4.1b: *Deflection profile of non-uniform Rayleigh beam under moving distributed loads for various values of axial force (N_o) and fixed values of foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc) and rotatory inertia correction factor (R_o).*

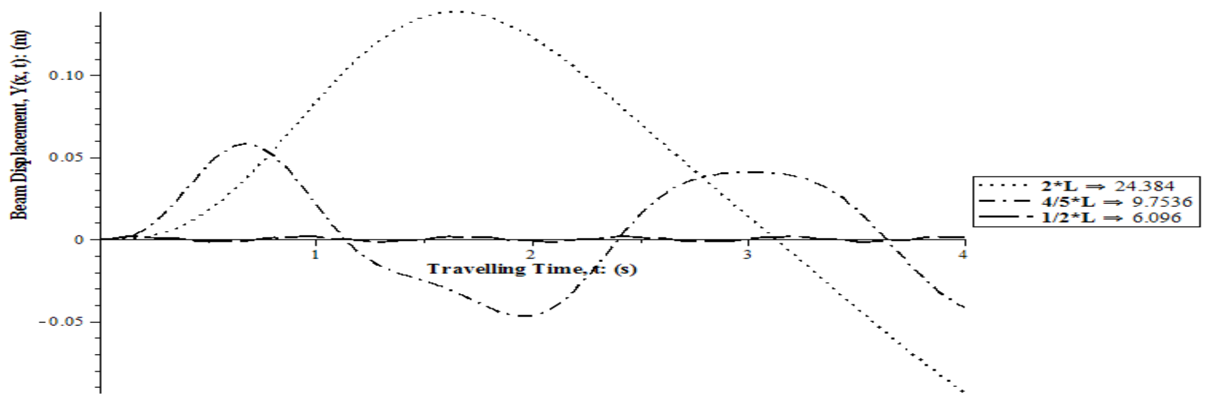

Figure 4.2a: *Deflection profile of non-uniform Rayleigh beam under moving constant distribution loads for fixed values of axial force (N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc) and rotatory inertia correction factor (R_o) when length L = 6.096m, L = 9.7536m and L = 24.394m.*

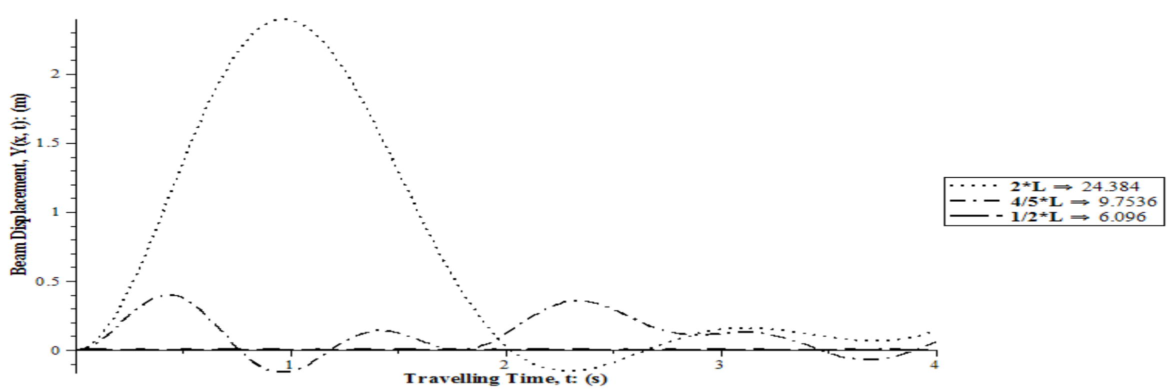

Figure 4.2b: *Deflection profile of non-uniform Rayleigh beam under moving distributed loads for fixed values of axial force (N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc) and rotatory inertia correction factor (R_o) when length L = 6.096m, L = 9.7536m and L = 24.394m.*

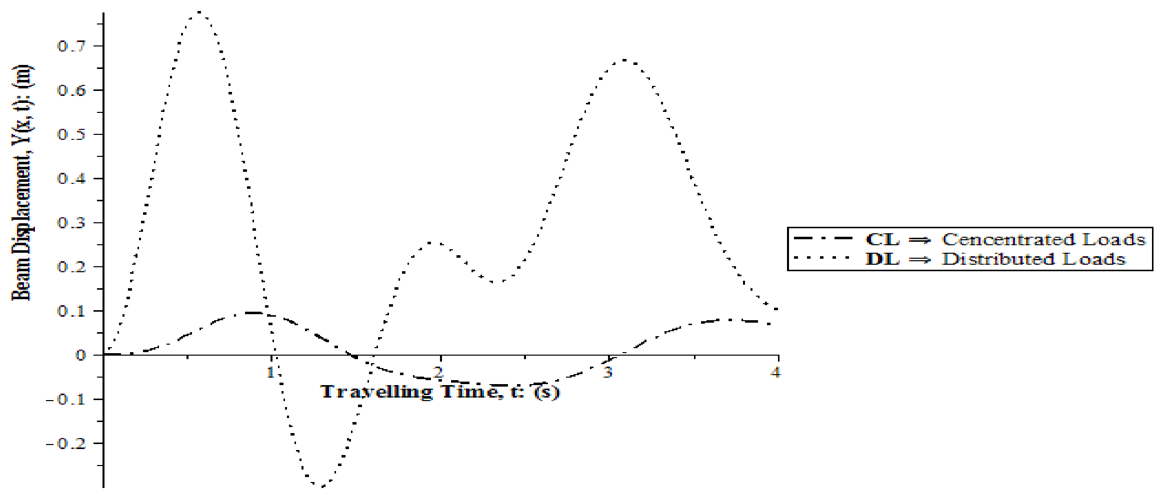

Figure 4.3: *Comparison of the deflection profiles between moving constant distribution and distributed loads for fixed values of structural parameters.*

Analysis

The analysis for the moving constant distribution and distributed load problems exhibited the following features:

1. ***Figure 4.1a & 4.1b:*** Shows the axial force influence on the dynamic response of simply-supported non-uniform Rayleigh beam resting on bi-parametric sub-grade under moving constant distribution and distributed loads. Depicts the effects of structural parameters such as axial force (N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc), and rotatory inertia (R_o) on the deflection profile to simply-supported non-uniform Rayleigh beam resting on bi-parametric foundation under moving constant distribution and distributed loads.

2. ***Figure 4.2a & 4.2b:*** Shows the influence of varying length on the deflection profile to the simply-supported non-uniform Rayleigh beam resting on bi-parametric foundation under moving constant distribution and distributed loads

3. ***Figures 4.3:*** Shows the comparison of the deflection profiles of simply-supported non-uniform Rayleigh beam resting on bi-parametric sub-grade under moving constant distribution loads to that under moving distributed loads.

5. Conclusion

From the plotted curves, it was observed that as the values of the structural parameters such as axial force(N_o), foundation stiffness (K_o), shear modulus (F_o), damping coefficient (Δc), and rotatory inertia correction factor (R_o) increases, the deflection profiles of the beam decreases for both moving constant distribution and distributed loads. Also, it was observed that the beam dynamic deflection to the moving distributed loads solution is greater than that of moving constant distribution loads for any values of the structural parameters. This shows that it will be tragic to use the reinforcement specified for the Rayleigh beam under moving constant distribution loads to maintain the Rayleigh beam under moving distributed loads, since the moving constant distribution or point loads are mere mathematical idealization for distributed loads of very high intensity which cannot be found in the real world. Hence, the moving distributed loads on the other hand practically provides more accurate model of the moving load problems as the loads are actually distributed over a small segment or the entire length of the non-uniform Rayleigh beam they traverse..

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